CSCI E-234 Introduction to Computer Graphics   HW3 - Written Part

Russell Lowke, Nov 10th 2004

Part 1 – Some transformations commute (5 points)

[from Hill, 5.2.20, p 228]

Show that uniform scaling commutes with rotation, in that the resulting transformation does not depend on the order in which the individual transformations are applied, and that rotation and non-uniform (a.k.a differential) scaling do not commute.

A scaling matrix such as,

```
[ 2, 0,  0 ]
[ 0, 2,  0 ]
[ 0, 0,  1 ]
```

when matrix multiplied with a rotation matrix, such as,

```
[ cos 30,  -sin 30,  0 ]
[ sin 30,   cos 30,  0 ]
[      0,        0,  1 ]
```

will rotate around the origin by 30 degrees, yielding the same transformation (commute) regardless of which matrix is applied first.

When the matrix multiplication is done by hand, the reasons for this become evident. Specifically, the scaling matrix is bilateral (reflects itself) along the diagonal, so it makes no difference if its columns or rows are calculated first in the matrix multiplication.

First I will evaluate the trigonometry out of the rotation matrix

```
[ cos 30,  -sin 30,  0 ]         [ 0.8660,     -0.5,   0 ]
[ sin 30,   cos 30,  0 ]    =    [    0.5,   0.8660,   0 ]
[      0,        0,  1 ]         [      0,        0,   1 ]
```

Now I calculate the matrix multiplication in both orders--obtaining the same result.

```
[ 2, 0,  0 ] [ 0.8660,     -0.5,   0 ]         [ 1.732,      -1, 0 ]
[ 0, 2,  0 ] [    0.5,   0.8660,   0 ]   =     [     1,  1.732, 0 ]
[ 0, 0,  1 ] [      0,        0,   1 ]         [     0,      0, 1 ]

[ 0.8660,     -0.5,   0 ] [ 2, 0,  0 ]         [ 1.732,      -1, 0 ]
[    0.5,   0.8660,   0 ] [ 0, 2,  0 ]   =     [     1,  1.732, 0 ]
[      0,        0,   1 ] [ 0, 0,  1 ]         [     0,      0, 1 ]
```

So it can be seen that scaling commutes with rotation due to the diagonally bilateral nature of the scaling matrix.

On the other hand, a translation matrix, such as,

```
[ 2, 0,  0 ]
[ 0, 2,  0 ]
[ 0, 0,  1 ]
```

when matrix multiplied with the rotation matrix,

```
[ 0.8660,     -0.5,   0 ]
[    0.5,   0.8660,   0 ]
[      0,        0,   1 ]
```

will yield different results (differential) depending on the order they are matrix multiplied, because neither of the two matrices is bilateral along its diagonal. As can be seen below,

```
[ 1, 0, -2 ] [ 0.8660,     -0.5,   0 ]         [ 0.866,   -0.5, -2 ]
[ 0, 1, -1 ] [    0.5,   0.8660,   0 ]   =     [   0.5,  0.866, -1 ]
[ 0, 0,  1 ] [      0,        0,   1 ]         [     0,      0,  1 ]

[ 0.8660,     -0.5,   0 ] [ 1, 0, -2 ]         [ 0.866,  -0.5, -1.232 ]
[    0.5,   0.8660,   0 ] [ 0, 1, -1 ]   =     [   0.5, 0.866, -1.866 ]
[      0,        0,   1 ] [ 0, 0,  1 ]         [     0,     0,      1 ]
```

So it can be seen that rotation does not commute with translation, due to neither the translation nor rotation matrices being bilateral along its diagonal.

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Part 2 – Find the reflected direction (5 points)

[from Hill, 4.3.13 p 160]

For a = (2, 3) and n = (-2, 1), find the direction of the reflection.

The equation for calculating a reflected (R) vector is,

$R = -a + 2(a \bullet n)n$

Where the dot product (•) for (a_1, a_2) and (b_1, b_2) is $a_1b_1 + a_2b_2$.
So the dot product of $(a \bullet n)$ is, $(2 * -2 + 3 * 1) = (-4 + 3) = -1$

Substituting into $R = -a + 2(a \bullet n)n$ we get,

$R_1 = -2 + 2 * -1 * -2 = 2$
$R_2 = -3 + 2 * -1 * 1 = -5$

So the reflected direction R = (2, -5) when a = (2, 3) and n = (-2, 1)

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Part 3 – Lengths of the incident and reflected vectors (5 points)

[from Hill, 4.3.14, p 160]

Using Equation (4.27) and properties of the dot product, show that $|r| = |a|$.

Looking at fig 4.14, reflection of a ray from a surface on page 160 of hill

$r = a - 2(a \bullet n)n$ where n is a unit vector in the direction of the normal from e.

$a = m + e$

while

$r = e - m$

substituting into $r = a - 2(a \bullet n)n$

$r = (m + e) - 2((m + e) \bullet n)n$

which will yield

$r = -(m + e)$
$r = -a$

so $|r|$ will equal $|a|$, r being the reflection of a.