CSCI E-234 Introduction to Computer Graphics HW2 - Written Part

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Part 1 – Finding matrices (5 points)

[from Hill, 5.2.18, p 228]

Give the explicit form of the three-by-three matrix representing each of the following transformations. Write down all the matrices in the right order before multiplying them together into the result matrix.

a. Scaling by a factor of 2 in the *x*-direction and then rotating about (2, 1) with theta = -30 degrees.

to scale by factor of 2 in the x-direction we use:

```
[ 2, 0, 0 ]  [ x ]
[ 0, 1, 0 ]  [ y ]
[ 0, 0, 1 ]  [ 1 ]
```

to rotate about 2,1 with theta = -30,
we must translate (2, 1) using:

```
[ 1, 0, 2 ]  [ x ]
[ 0, 1, 1 ]  [ y ]
[ 0, 0, 1 ]  [ 1 ]
```

rotate around theta using:

```
[ -cos 30, sin  30,  0 ]  [ x ]
[ -sin 30, -cos 30,  0 ]  [ y ]
[       0,       0,  1 ]  [ 1 ]
```

which is:

```
[ -0.8660, 0.5,   0 ]  [ x ]
[ -0.5, -0.8660,  0 ]  [ y ]
[   0,        0,  1 ]  [ 1 ]
```

and translate back (-2, -1) using:

```
[ 1, 0, -2 ]  [ x ]
[ 0, 1, -1 ]  [ y ]
[ 0, 0,  1 ]  [ 1 ]
```

…but when calculating the resultant matrix, all matrices are applied in a “first in last out” reverse order, giving:

```
[ 1, 0, -2 ] [ -0.8660,  0.5,  0 ] [ 1, 0, 2 ] [ 2, 0, 0 ]   [ x ]
[ 0, 1, -1 ] [ -0.5, -0.8660,  0 ] [ 0, 1, 1 ] [ 0, 1, 0 ]   [ y ]
[ 0, 0,  1 ] [   0,        0,  1 ] [ 0, 0, 1 ] [ 0, 0, 1 ]   [ 1 ]
```

so the resultant transformation matrix is:

```
[ -1.732,    0.5, -3.232 ]   [ x ]
[   -1.0, -0.866, -2.866 ]   [ y ]
[      0,      0,      1 ]   [ 1 ]
```

b. Scaling about (2, 3) and following by translation through (1, 1).

to scale about 2,3,
we must translate (2, 3) using:

```
[ 1, 0, 2 ] [ x ]
[ 0, 1, 3 ] [ y ]
[ 0, 0, 1 ] [ 1 ]
```

then scale, in this case by x 2, using:

```
[ 2, 0,  0 ] [ x ]
[ 0, 2,  0 ] [ y ]
[ 0, 0,  1 ] [ 1 ]
```

and translate back (-2, -3) using:

```
[ 1, 0, -2 ] [ x ]
[ 0, 1, -3 ] [ y ]
[ 0, 0,  1 ] [ 1 ]
```

then following by translation through 1,1

```
[ 1, 0, 1 ]  [ x ]
[ 0, 1, 1 ]  [ y ]
[ 0, 0, 1 ]  [ 1 ]
```

…but when calculating the resultant matrix, all matrices are applied in a “first in last out” reverse order, giving:

```
[ 1, 0, 1 ] [ 1, 0, -2 ] [ 2, 0,  0 ] [ 1, 0, 2 ]  [ x ]
[ 0, 1, 1 ] [ 0, 1, -3 ] [ 0, 2,  0 ] [ 0, 1, 3 ]  [ y ]
[ 0, 0, 1 ] [ 0, 0,  1 ] [ 0, 0,  1 ] [ 0, 0, 1 ]  [ 1 ]
```

so the resultant transformation matrix is:

```
[ 2, 0, 3 ] [ x ]
[ 0, 2, 4 ] [ y ]
[ 0, 0, 1 ] [ 1 ]
```

c. Shearing by 30% in *x*, scaling by 2 in *x*, and then rotating about (1, 1) through 30 degrees.

shear by 30% in x using:

```
[ 1, .3,  0 ]  [ x ]
[ 0,  1,  0 ]  [ y ]
[ 0,  0,  1 ]  [ 1 ]
```

scale by factor of 2 in the x-direction using:

```
[ 2, 0, 0 ]  [ x ]
[ 0, 1, 0 ]  [ y ]
[ 0, 0, 1 ]  [ 1 ]
```

to rotate about 1,1 with theta = 30,
we must translate (1, 1) using:

```
[ 1, 0, 1 ] [ x ]
[ 0, 1, 1 ] [ y ]
[ 0, 0, 1 ] [ 1 ]
```

rotate with theta = 30,

```
[ cos 30,  -sin 30,   0 ] [ x ]
[ sin 30,  cos  30,   0 ] [ y ]
[     0,        0,    1 ] [ 1 ]
```

which is:

```
[ 0.8660,     -0.5,   0 ] [ x ]
[    0.5,   0.8660,   0 ] [ y ]
[      0,        0,   1 ] [ 1 ]
```

and translate back (-1, -1) using:

```
[ 1, 0, -1 ]   [ x ]
[ 0, 1, -1 ]   [ y ]
[ 0, 0,  1 ]   [ 1 ]
```

…but when calculating the resultant matrix, all matrices are applied in a “first in last out” reverse order, giving:

```
[1, 0, -1 ] [ 0.8660,   -0.5,  0 ] [ 1, 0, 1 ] [ 2, 0, 0 ] [ 1, .3,  0 ] [ x ]
[0, 1, -1 ] [  0.5,   0.8660,  0 ] [ 0, 1, 1 ] [ 0, 1, 0 ] [ 0,  1,  0 ] [ y ]
[0, 0,  1 ] [    0,        0,  1 ] [ 0, 0, 1 ] [ 0, 0, 1 ] [ 0,  0,  1 ] [ 1 ]
```

so the resultant transformation matrix is:

```
[ 1.732,  0.0196,  -0.634 ] [ x ]
[     1,   1.166,   0.366 ] [ y ]
[     0,       0,       1 ] [ 1 ]
```

Part 2 Perform some conversions (5 points)
[from Hill, 12.4.2, p 688]

Find (*H, L, S*) for each of the following cases, and explain why the result is reasonable:

Hue can be associated with an angle between 0 and 360, with 0 at red.
Lightness is a value from 0 to 1
Saturation mapped to a radial distance.

a) (*r, g, b*) = (.2, .8, .1);

(h, l, s) = (111.429, 0.45, 0.777778);
This color is lime green. Reasonable as green is about 1/3rd of the way along the hue spectrum/range. Lime is very saturated, so a high value here makes sense.

b) (*r, g, b*) = (0, 0, .8);

(h, l, s) = (240, 4, -1.33333);
This color is navy blue. Reasonable as blue is about 2/3rds along the hue spectrum/range. A very high (negative) saturation makes this a very intense blue.

c) (*r, g, b*) = (1, 1, 1);

(h, l, s) = (0, 1, 0);
This color is white. The Hue and Saturation are zero, while the luminosity is on full 100%, bright white.

d) (*r, g, b*) = (0, .7, .7);

(h, l, s) = (180, 0.35, 1);
This is a teal color, slightly lower on the hue range than blue. The color is less light/bright, but has a very high/full saturation of 1.